PHYC 581: High Energy Astrophysics
Fall 2018

Lecture 15: 10/10/2018

Comptonization (Cont'd):

The question is how to describe the evolution of the spectrum toward equilibrium as a result of Comptonization. In the non-relativistic limit, the equation that describes the evolution is called the "Kompaneets equation". This equation takes account of the interchange of energy between photons and electrons in both direction, as well as induced effects that become important when the photon occupation number f is greater than 1.

The Kompaneets equation is:

$$\frac{\partial f}{\partial y} = \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \left(f + f^2 + \frac{\partial f}{\partial x} \right) \right]$$

Here:

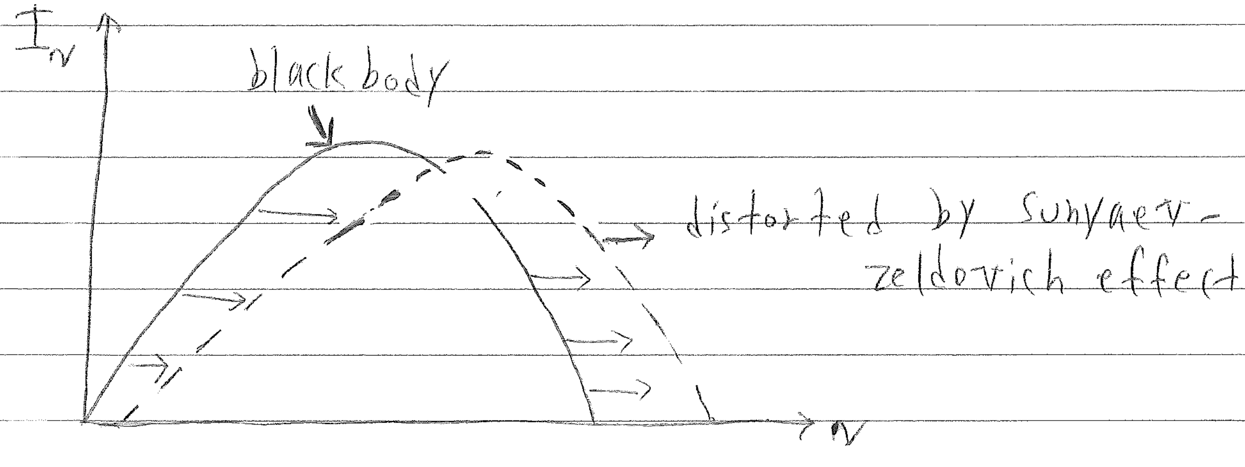
$$dy = \frac{kT}{m_e c^2} \sigma_T n_e \, dl \, , \qquad x = \frac{h\nu}{kT}$$

The first term on the right-hand side of the equation, which is $\alpha \frac{\partial f}{\partial x}$, represents the cooling of the photons by the recoil effect. The second term, which is quadratic in f, describes the effect of induced Compton scattering and becomes important when $f \gtrsim O(1)$. The last term, which involves $\frac{\partial^2 f}{\partial x^2}$, represents the diffusion of photons along the frequency as a result of (induced) Compton scattering. It is easy to see that the right-hand side vanishes, as expected, in thermal equilibrium when $f = [\exp(x-\nu)-1]^{-1}$.

In general, the solutions to the Kompaneets equation have to be found numerically, but some useful limiting cases exist.

Sunyaev-Zeldovich Effect:

A very important application of the Kompaneets equation is in describing distortions of the spectrum of the CMB radiation. This happens if the CMB photons are scattered by hot electrons in regions of very hot ionized gas as they propagate toward us. The largest effect is expected in the direction of those rich galaxy clusters that possess large amounts of hot gas, for example, the Perseus cluster. The effect is known as the "Sunyaev-Zeldovich" effect. The Sunyaev-Zeldovich effect results in the depletion of the Rayleigh-Jeans part of the blackbody spectrum and the population of the Wien tail. This can be also verified by examining the Kompaneets equation for $h\nu \ll kT_\gamma$ and $h\nu \gg kT_\gamma$.



The change in the brightness temperature of the CMB radiation in the Rayleigh-Jeans part is proportional to n_eT (n_e being the electron number density). It therefore provides an estimate of the pressure of the hot intergalactic gas.

The Sunyaev-Zeldovich effect has a very important cosmological application. The temperature of the hot intergalactic gas in the clusters can be determined from the shape of the Bremsstrahlung spectrum as mentioned before. For example, in the case of the Perseus cluster we saw that $T = 7.5\times10^7$ K is found, which corresponds to an energy of 6.5 keV. One can use this in combination with the Sunyaev-Zeldovich effect to find the electron density and the physical size of the emitting gas cloud. By measuring its angular extent, the distance to the cluster can be measured. If the redshift of the cluster is known, one can measure the Hubble expansion rate at the present time.

The change in the brightness temperature of the CMB (in the Rayleigh-Jeans part of the spectrum) in the direction of rich galaxy clusters is $O(mK)$. This is compatible with the average energy gain $\propto \frac{kT}{m_e c^2}$ by the photons in a single Compton collision for temperatures $T \sim 10^7 - 10^8$ K. It also underlines the fact that Compton emission is more important for non-thermal sources.

The Sunyaev-Zeldovich effect discussed above, which is due to scattering off hot electrons that have random thermal motion, is called "Thermal Sunyaev-Zeldovich Effect". When averaged over the photon-electron incident angle, the photon energy gain is proportional to $(\frac{v}{c})^2$ (v being the thermal velocity). For $T \sim 10^8$ K, we have $(\frac{v}{c})^2 \sim \frac{1}{50}$. There also exists "Kinetic Sunyaev-Zeldovich Effect", which occurs if the photons scatter off electrons in an object with a bulk flow. The photon energy gain in this case is proportional to $\frac{v}{c}$, where v is the bulk velocity. Although a linear effect, the kinetic Sunyaev-Zeldovich effect is subdominant unless the bulk velocity is high enough ($\frac{v}{c} \sim O(\frac{1}{50})$ or so).

The kinetic Sunyaev-Zeldovich effect in an individual object was first observed in 2012 (and confirmed in 2013). It was detected by observing a high-speed component of the MACS J0717.5+3745 galaxy cluster. This cluster has a total mass of $>$ 1000 times of our galaxy, and includes four subclusters three of which are stationary. The fourth subcluster is traveling at about 3,000 km/s.

Observations of this subcluster at two frequencies (140 GHz and 268 GHz) was not compatible with the thermal Sunyaev-Zeldovich effect. However, when the kinetic Sunyaev-Zeldovich effect was taken into account, the predictions matched the observations. Prior to this, the best indication of the kinetic Sunyaev-Zeldovich effect came from multiple galaxies and galaxy clusters, originally detected by the Atcama Cosmology Telescope (ACT) and the Sloan Digital Sky Survey (SDSS).